MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.
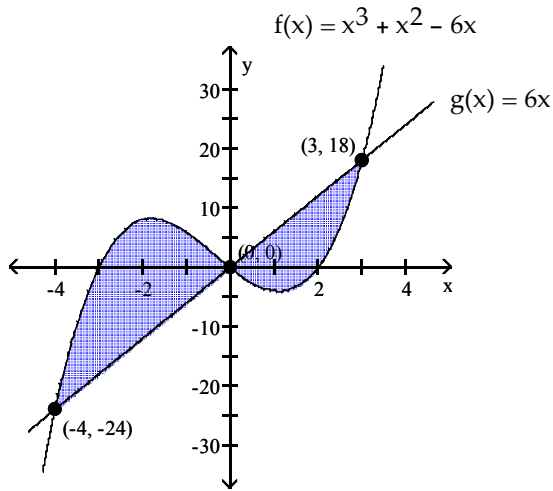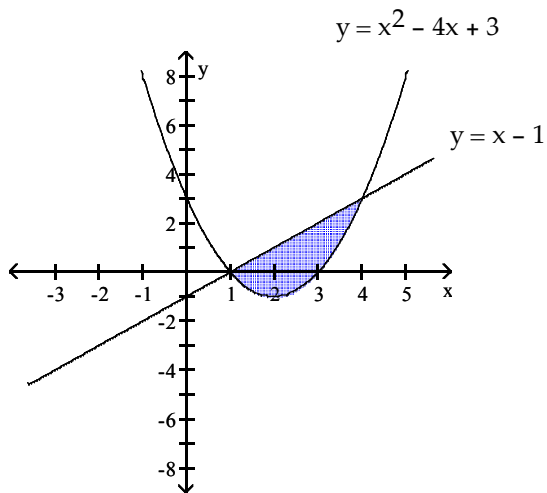
Find the area of the shaded region.

1) $f(x) = x^3 + x^2 - 6x$ 1) _____

A) $\dfrac{343}{12}$ B) $\dfrac{81}{12}$ C) $\dfrac{768}{12}$ D) $\dfrac{937}{12}$
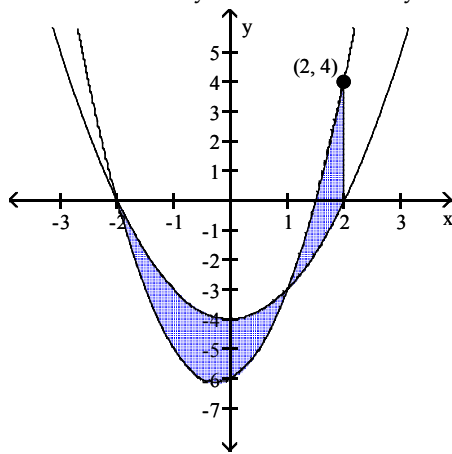
2) $y = x^2 - 4x + 3$ 2) _____

A) $\dfrac{9}{2}$ B) $\dfrac{41}{6}$ C) $\dfrac{25}{6}$ D) 3

3) $y = 2x^2 + x - 6$ $y = x^2 - 4$

(2, 4)

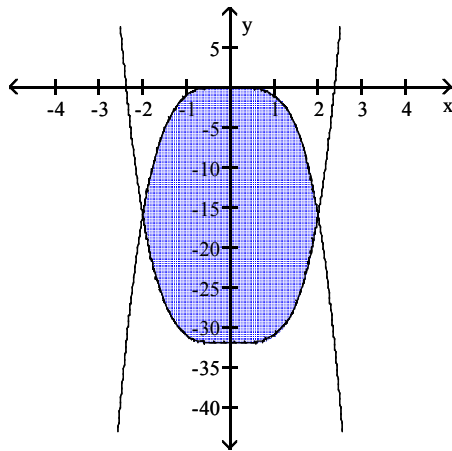A) $\dfrac{19}{3}$ B) $\dfrac{9}{2}$ C) $\dfrac{8}{3}$ D) $\dfrac{11}{6}$
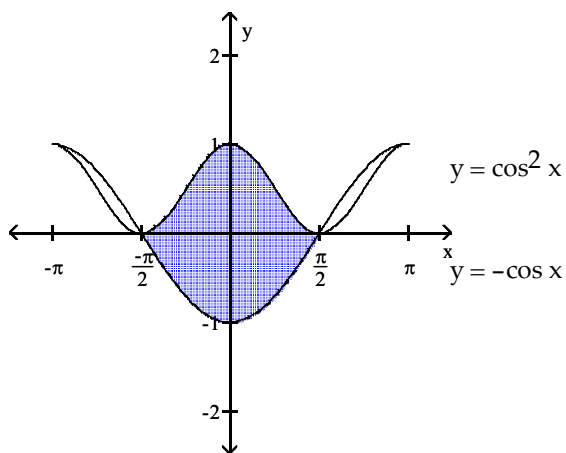
4) $y = x^4 - 32$

$y = -x^4$

A) $\dfrac{516}{5}$ B) $\dfrac{256}{5}$ C) $\dfrac{2816}{5}$ D) $\dfrac{512}{5}$

5)

$y = \cos^2 x$

$y = -\cos x$

A) 2 B) 2 + π C) $2 - \dfrac{\pi}{2}$ D) $2 + \dfrac{\pi}{2}$

Find the area enclosed by the given curves.

6) $y = 2x - x^2$, $y = 2x - 4$

    A) $\dfrac{32}{3}$ B) $\dfrac{31}{3}$ C) $\dfrac{34}{3}$ D) $\dfrac{37}{3}$

7) $y = x$, $y = x^2$

    A) $\dfrac{1}{3}$ B) $\dfrac{1}{6}$ C) $\dfrac{1}{2}$ D) $\dfrac{1}{12}$

8) $y = -4\sin x$, $y = \sin 2x$, $0 \le x \le \pi$

    A) 4 B) 16 C) 8 D) $\dfrac{1}{2}$

9) $y = \csc^2 x$, $y = \cot^2 x$, $x = \dfrac{\pi}{4}$, and $x = \dfrac{3\pi}{4}$

    A) $\dfrac{\pi}{4}$ B) $\dfrac{3\pi}{4}$ C) π D) $\dfrac{\pi}{2}$

10) Find the area of the region in the first quadrant bounded on the left by the line $y = \dfrac{\pi}{6}$ and on the

    right by the curves $y = \tan^2 x$ and $y = \cot^2 x$. (Round to four decimal places.)
    A) 4.3094 B) 0.4126 C) 0.5858 D) 0.3094

Find the volume of the described solid.

11) The solid lies between planes perpendicular to the x–axis at $x = 0$ and $x = 9$. The cross sections
    perpendicular to the x–axis between these planes are squares whose bases run from the parabola
    $y = -3\sqrt{x}$ to the parabola $y = 3\sqrt{x}$.
    A) 243 B) 1440 C) 729 D) 1458

12) The base of the solid is the disk $x^2 + y^2 \leq 4$. The cross sections by planes perpendicular to the
y–axis between $y = -2$ and $y = 2$ are isosceles right triangles with one leg in the disk.

A) $\dfrac{128}{3}$ B) $\dfrac{64}{3}$ C) $\dfrac{80}{3}$ D) $\dfrac{16}{3}$

Find the volume of the solid generated by revolving the shaded region about the given axis.

13) About the x–axis
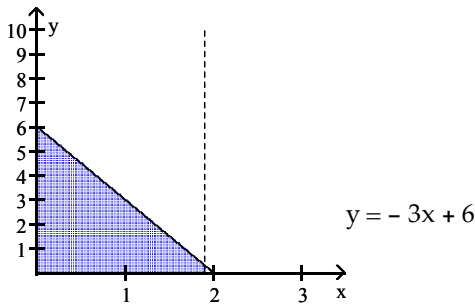
$y = -3x + 6$

A) 168π B) 24π C) 18π D) 48π

14) About the x–axis

$y = 4\sec x$

A) 4π B) 16π C) 8π D) 14π

15) About the y–axis

$y = \sqrt{3x}$
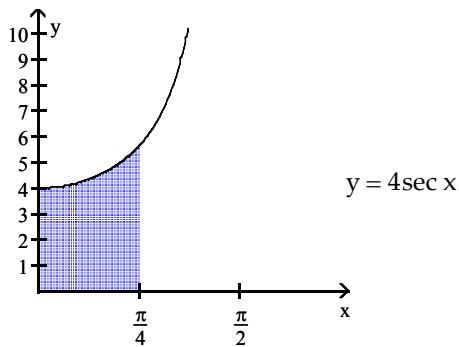
A) $\dfrac{243}{5}\pi$ B) $\dfrac{27}{5}\pi$ C) 3π D) 18π

16) About the y-axis   16) _____

$$x = 2\tan\left(\frac{y}{5}\right)$$

A) $5\pi^2 - 10\pi$ B) 20π C) $10\pi^2 + 5\pi$ D) $10\pi^2 - 20\pi$

17) About the x-axis   17) _____

$$y = 9 - x^2$$

A) $\dfrac{162}{5}\pi$ B) $\dfrac{567}{5}\pi$ C) 9π D) $\dfrac{648}{5}\pi$

Find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.
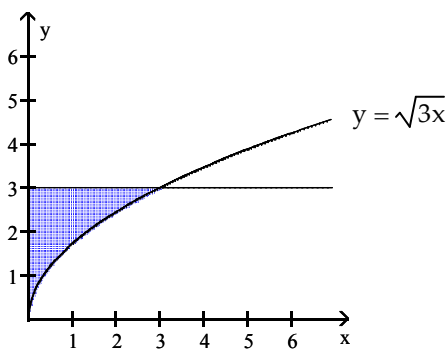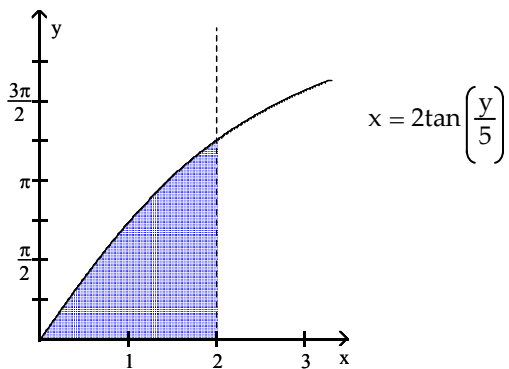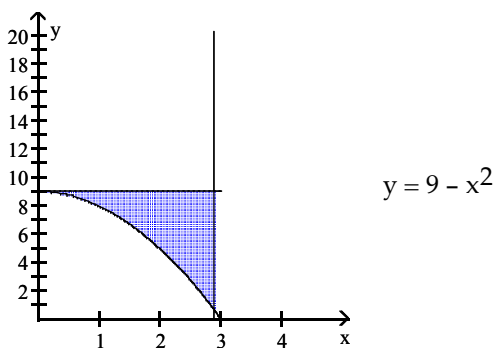
18) $y = x$, $y = 0$, $x = 3$, $x = 4$ 18) _____

   A) $\dfrac{1}{3}\pi$ B) $\dfrac{37}{3}\pi$ C) $\dfrac{7}{2}\pi$ D) $\dfrac{25}{2}\pi$

19) $y = x^2$, $y = 0$, $x = 0$, $x = 5$ 19) _____

   A) 625π B) $\dfrac{625}{4}\pi$ C) $\dfrac{3125}{4}\pi$ D) $\dfrac{125}{3}\pi$

20) $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, $x = 2$ 20) _____

   A) $\dfrac{1}{4}\pi$ B) $\pi\ln 2$ C) $\dfrac{3}{2}\pi$ D) $\dfrac{1}{2}\pi$

21) $y = \sqrt{81 - x^2}$, $y = 0$, $x = 0$, $x = 9$ 21) _____
   A) 972π B) 18π C) 486π D) 324π

5

22) $y = 7\csc x$, $y = 0$, $x = \dfrac{\pi}{4}$, $x = \dfrac{3\pi}{4}$

    A) 49π B) 98π C) 147π D) 14π

22) _____

23) $y = 6x$, $y = 6$, $x = 0$

    A) 24π B) 3π C) 18π D) 12π

23) _____

24) $y = \sqrt{5x}$, $y = 5$, $x = 0$

    A) $\dfrac{125}{2}\pi$ B) $\dfrac{125}{3}\pi$ C) 50π D) $\dfrac{125}{4}\pi$

24) _____

25) $y = x^2 + 5$, $y = 3x + 5$

    A) $\dfrac{693}{5}\pi$ B) $\dfrac{387}{5}\pi$ C) $\dfrac{279}{10}\pi$ D) 27π

25) _____

26) $y = \sqrt{\sin 4x}$, $y = 1$, $x = 0$ to $x = \dfrac{\pi}{8}$

    A) $\dfrac{\pi^2}{8} - \dfrac{\pi}{4}$ B) $\dfrac{\pi^2}{8} + \pi$ C) $\dfrac{\pi^2}{8} - \pi$ D) $\dfrac{\pi}{8} - \dfrac{1}{4}$

26) _____

27) $y = 9\cos(\pi x)$, $y = 9$, $x = -0.5$, $x = 0.5$

    A) 162π B) 27π C) 81π D) $\dfrac{81}{2}\pi$

27) _____

Find the volume of the solid generated by revolving the region about the given line.

28) The region bounded above by the line $y = 9$, below by the curve $y = 9 - x^2$, and on the right by the line $x = 3$, about the line $y = 9$

    A) $\dfrac{648}{5}\pi$ B) 9π C) $\dfrac{243}{5}\pi$ D) $\dfrac{567}{5}\pi$

28) _____

29) The region in the first quadrant bounded above by the line $y = 3$, below by the curve $y = \sqrt{3x}$, and on the left by the y–axis, about the line $y = 3$

    A) $\dfrac{9}{2}\pi$ B) 3π C) $\dfrac{27}{5}\pi$ D) $\dfrac{27}{2}\pi$

29) _____

30) The region bounded above by the line $y = 2$, below by the curve $y = 2\cos(\pi x)$, on the left by the line $x = -0.5$, and on the right by the line $x = 0.5$, about the line $y = 2$

    A) 2π B) 6π C) $6\pi - 16$ D) $12\pi - 8$

30) _____

31) The region in the first quadrant bounded above by the line $6x + y = 12$, below by the x–axis, and on the left by the y–axis, about the line $x = -2$

    A) 10π B) 128π C) 96π D) 64π

31) _____

32) The region in the first quadrant bounded above by the line $y = 5x^3$, below by x-axis, and on the right by the line $x = 1$, about the line $y = -1$

A) $\dfrac{85}{14}\pi$ B) $\dfrac{45}{14}\pi$ C) $\dfrac{85}{7}\pi$ D) $\dfrac{32}{7}\pi$

32) _____

Find the volume of the solid generated by revolving the region about the y-axis.

33) The region enclosed by $x = \dfrac{y^2}{2}$, $x = 0$, $y = -2$, $y = 2$

A) $\dfrac{8}{5}\pi$ B) $\dfrac{64}{5}\pi$ C) $\dfrac{8}{3}\pi$ D) $\dfrac{16}{5}\pi$

33) _____

34) The region enclosed by $x = \dfrac{5}{y}$, $x = 0$, $y = 1$, $y = 4$

A) $\dfrac{75}{4}\pi$ B) $\dfrac{75}{16}\pi$ C) $\dfrac{125}{4}\pi$ D) $\dfrac{15}{4}\pi$

34) _____

35) The region enclosed by $x = \sqrt{\sin 3y}$, $0 \le y \le \dfrac{\pi}{6}$, $x = 0$

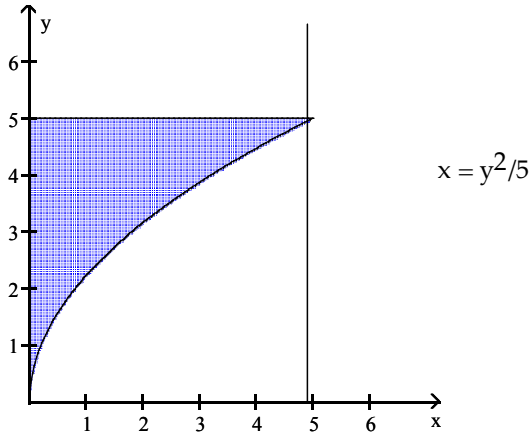A) 6π B) $\dfrac{\pi}{6}$ C) 3π D) $\dfrac{\pi}{3}$

35) _____

36) The region enclosed by the triangle with vertices $(0, 0)$, $(4, 0)$, $(4, 2)$

A) 64π B) $\dfrac{64}{3}\pi$ C) $\dfrac{16}{3}\pi$ D) $\dfrac{32}{3}\pi$

36) _____

37) The region in the first quadrant bounded on the left by $y = x^3$, on the right by the line $x = 3$, and below by the x-axis

A) $\dfrac{81}{4}\pi$ B) $\dfrac{243}{5}\pi$ C) $\dfrac{729}{5}\pi$ D) $\dfrac{486}{5}\pi$

37) _____

Use the shell method to find the volume of the solid generated by revolving the shaded region about the indicated axis.

38) About the x–axis   38) _____

$x = y^2/5$

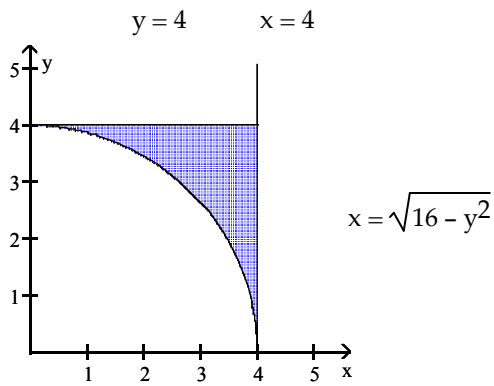A) $\dfrac{125}{3}\pi$ B) 50π C) $\dfrac{125}{2}\pi$ D) $\dfrac{125}{4}\pi$

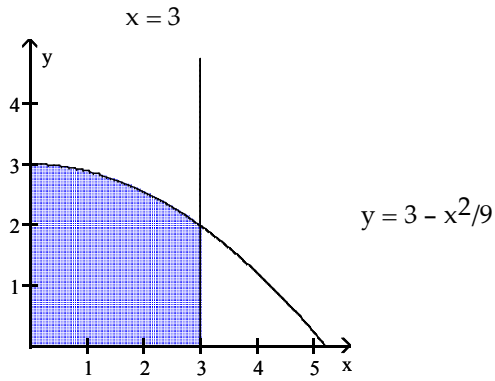39) About the x–axis   39) _____

$y = 4$ $x = 4$

$x = \sqrt{16 - y^2}$

A) 64π B) $\dfrac{128}{3}\pi$ C) $\dfrac{32}{3}\pi$ D) $\dfrac{64}{3}\pi$

40) About the y-axis

$x = 3$

$y = 3 - x^2/9$

A) $\dfrac{45}{4}\pi$　　　　　B) 18π　　　　　C) $\dfrac{45}{2}\pi$　　　　　D) 27π

40) _____

41) About the y-axis

$y = 3x - x^2$

A) $\dfrac{81}{8}\pi$　　　　　B) $\dfrac{27}{2}\pi$　　　　　C) $\dfrac{81}{4}\pi$　　　　　D) $\dfrac{27}{4}\pi$

41) _____
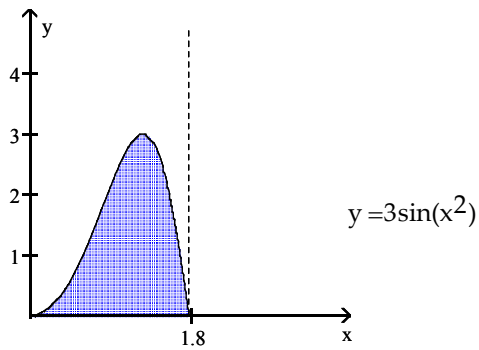
42) About the y-axis

$y = 3\sin(x^2)$

1.8

A) 3π　　　　　B) 9π　　　　　C) 12π　　　　　D) 6π

42) _____

9

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the y–axis.

43) $y = 8x$, $y = -\dfrac{x}{8}$, $x = 1$ 43) _____

    A) $\dfrac{43}{8}\pi$ B) $\dfrac{65}{24}\pi$ C) $\dfrac{65}{12}\pi$ D) 65π

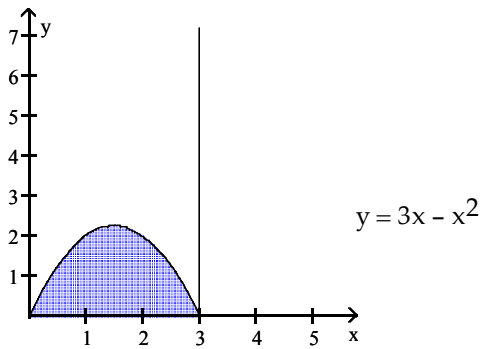44) $y = 2x^2$, $y = 2\sqrt{x}$ 44) _____
    A) $\dfrac{3}{10}\pi$ B) $\dfrac{3}{2}\pi$ C) $\dfrac{3}{5}\pi$ D) $\dfrac{6}{5}\pi$

45) $y = 32 - x^2$, $y = x^2$, $x = 0$ 45) _____
    A) 256π B) 128π C) 64π D) 512π

46) $y = \dfrac{4}{\sqrt{x}}$, $y = 0$, $x = 1$, $x = 16$ 46) _____

    A) 168π B) $\dfrac{1040}{3}\pi$ C) 336π D) $\dfrac{1024}{3}\pi$

47) $y = \dfrac{4}{x}$, $y = 0$, $x = 2$, $x = 4$ 47) _____

    A) 24π B) 8π C) 16π D) 12π

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the x–axis.

48) $x = 6\sqrt{y}$, $x = -6y$, $y = 1$ 48) _____
    A) 12π B) 16π C) $\dfrac{22}{5}\pi$ D) $\dfrac{44}{5}\pi$

49) $x = 9y - y^2$, $x = 0$ 49) _____
    A) 2187π B) $\dfrac{2187}{4}\pi$ C) $\dfrac{243}{2}\pi$ D) $\dfrac{2187}{2}\pi$

50) $y = 6x$, $y = 12x$, $y = 6$ 50) _____
    A) 6π B) 12π C) 18π D) 36π

51) $y = 5x^2$, $y = 5\sqrt{x}$ 51) _____
    A) $\dfrac{15}{4}\pi$ B) $\dfrac{15}{2}\pi$ C) $\dfrac{75}{2}\pi$ D) $\dfrac{3}{2}\pi$

52) $x = 5y^2$, $x = 5\sqrt[3]{y}$ 52) _____
    A) $\dfrac{25}{14}\pi$ B) $\dfrac{25}{28}\pi$ C) $\dfrac{5}{3}\pi$ D) $\dfrac{15}{2}\pi$

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves about the given lines.

53) $y = 2x$, $y = 0$, $x = 2$; revolve about the x–axis   53) _____

   A) $\dfrac{16}{3}\pi$ B) $\dfrac{32}{3}\pi$ C) $\dfrac{16}{3}$ D) $\dfrac{64}{3}\pi$

54) $y = 9 - x^2$, $y = 9$, $x = 3$; revolve about the line $y = 9$ 54) _____

   A) $\dfrac{567}{5}\pi$ B) 9π C) $\dfrac{243}{5}\pi$ D) $\dfrac{648}{5}\pi$
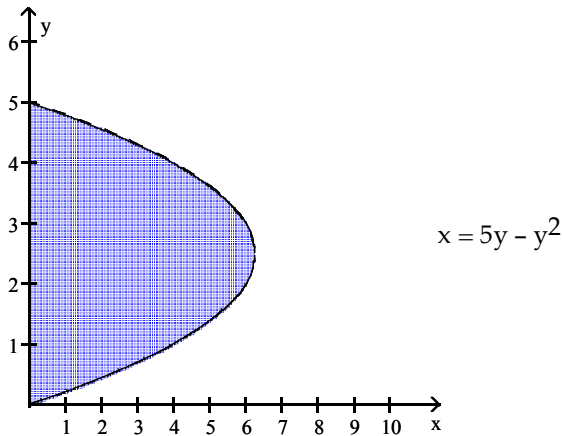
Use the shell method to find the volume of the solid generated by revolving the shaded region about the indicated line.

55) About the line $y = 5$ 55) _____

$x = 5y - y^2$
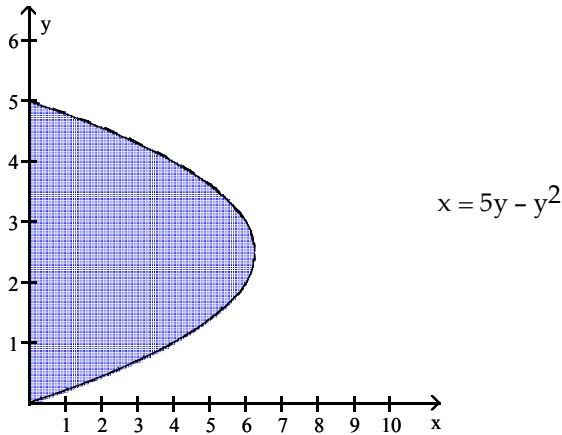
   A) $\dfrac{125}{6}\pi$ B) $\dfrac{625}{6}\pi$ C) $\dfrac{625}{3}\pi$ D) $\dfrac{625}{12}\pi$

56) About the line $y = -1$ 56) _____
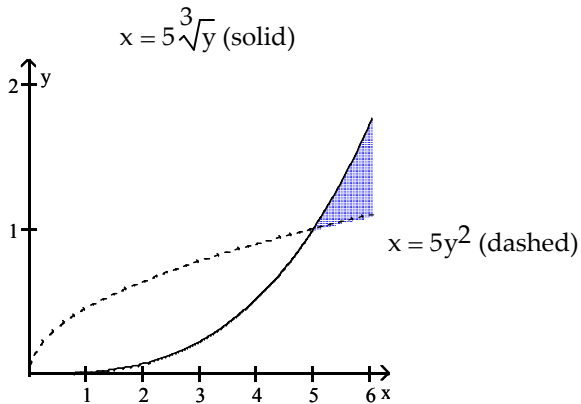

$x = 5y - y^2$

   A) $\dfrac{875}{12}\pi$ B) $\dfrac{625}{6}\pi$ C) $\dfrac{875}{2}\pi$ D) $\dfrac{875}{6}\pi$
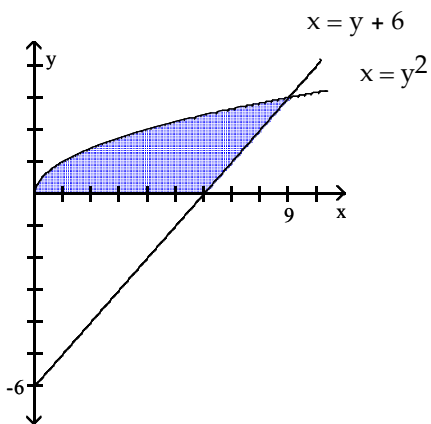
57) About the line y = –1

$x = 5\sqrt[3]{y}$ (solid)

$x = 5y^2$ (dashed)

57) _____

A) $\dfrac{125}{21}\pi$ B) $\dfrac{25}{14}\pi$ C) $\dfrac{250}{21}\pi$ D) $\dfrac{50}{21}\pi$
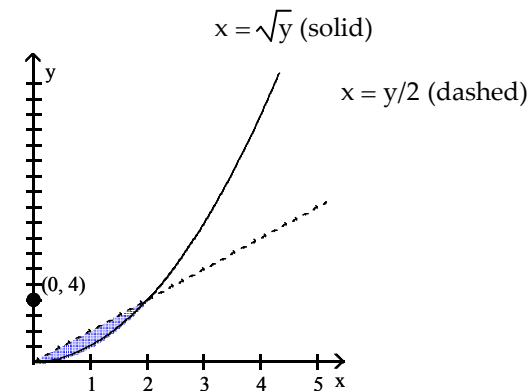
58) About the line y = – 6

$x = y + 6$

$x = y^2$

58) _____

A) 192π B) $\dfrac{63}{2}\pi$ C) $\dfrac{387}{2}\pi$ D) $\dfrac{387}{4}\pi$

59) About the line y = 4

$x = \sqrt{y}$ (solid)

$x = y/2$ (dashed)

(0, 4)

59) _____

A) $\dfrac{224}{15}\pi$ B) $\dfrac{16}{5}\pi$ C) $\dfrac{32}{5}\pi$ D) $\dfrac{64}{15}\pi$

12

Find the volume of the solid generated by revolving the region about the given axis. Use the shell or washer method.

60) The triangle with vertices (0, 0), (0, 2), and (2, 2) about the line $x = 2$ 60) _____

    A) $\dfrac{20}{3}\pi$ B) $\dfrac{4}{3}\pi$ C) $\dfrac{8}{3}\pi$ D) $\dfrac{16}{3}\pi$

61) The region bounded by $y = 7\sqrt{x}$, $y = 7$, and $x = 0$ about the line $y = 7$ 61) _____

    A) $\dfrac{49}{12}\pi$ B) $\dfrac{49}{3}\pi$ C) $\dfrac{49}{6}\pi$ D) $\dfrac{49}{2}\pi$

62) The region bounded by $y = 7\sqrt{x}$, $y = 7$, and $x = 0$ about the y-axis 62) _____

    A) $\dfrac{14}{3}\pi$ B) $\dfrac{7}{5}\pi$ C) $\dfrac{7}{3}\pi$ D) $\dfrac{7}{10}\pi$

63) The region bounded by $y = 5x - x^2$ and $y = x$ about the line $x = 4$ 63) _____

    A) $\dfrac{64}{3}\pi$ B) 64π C) 32π D) $\dfrac{128}{3}\pi$

64) The region in the first quadrant bounded by $x = 2y - y^2$ and the y-axis about the x-axis 64) _____

    A) 2π B) $\dfrac{8}{3}\pi$ C) $\dfrac{4}{3}\pi$ D) 4π
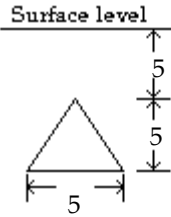
Solve the problem.

65) A bead is formed from a sphere of radius 3 by drilling through a diameter of the sphere with a drill bit of radius 1. Find the volume of the bead. 65) _____

    A) 36π B) 18π C) $\dfrac{56}{3}\pi$ D) $\dfrac{28}{3}\pi$

66) An auxiliary fuel tank for a helicopter is shaped like the surface generated by revolving the curve 66) _____

$y = 1 - \dfrac{x^2}{9}$, $-3 \le x \le 3$, about the x-axis (dimensions are in feet). How many cubic feet of fuel will the tank hold to the nearest cubic foot?

    A) 13   B) 3   C) 5   D) 10

67) The spring of a spring balance is 7.0 in. long when there is no weight on the balance, and it is 7.5 in. long with 4.0 lb hung from the balance. How much work is done in stretching it from 7.0 in. to a length of 13.5 in.? 67) _____

    A) 2.6 in.-lb   B) 26 in.-lb   C) 170 in.-lb   D) 530 in.-lb

68) It took 1920 J of work to stretch a spring from its natural length of 5 m to a length of 7 m. Find the spring's force constant. 68) _____

    A) 480 N/m   B) 1440 N/m   C) 3840 N/m   D) 960 N/m

69) It takes a force of 12,000 lb to compress a spring from its free height of 12 in. to its fully compressed height of 7 in. How much work does it take to compress the spring the first inch? 69) _____

    A) 600 in.-lb   B) 2400 in.-lb   C) 120,000 in.-lb   D) 1200 in.-lb

70) A force of 4 N will stretch a rubber band 5 cm. Assuming Hooke's law applies, how much work is   70) _____
done on the rubber band by a 12 N force?
   A) 0.1 J   B) 9000 J   C) 0.3 J   D) 0.9 J

71) Find the work done in winding up a 150–ft cable that weighs 4.00 lb/ft.   71) _____
   A) 1200 ft·lb   B) 11,300 ft·lb   C) 135,000 ft·lb   D) 45,000 ft·lb

72) The gravitational force (in lb) of attraction between two objects is given by $F = k/x^2$, where x is the   72) _____
distance between the objects. If the objects are 15 ft apart, find the work required to separate them
until they are 75 ft apart. Express the result in terms of k.

   A) $\frac{4}{75} k$ B) $\frac{1}{60} k$ C) $\frac{1}{5} k$ D) $\frac{1}{1125} k$

73) A rescue cable attached to a helicopter weighs 2 lb/ft. A 200–lb man grabs the end of the rope and   73) _____
is pulled from the ocean into the helicopter. How much work is done in lifting the man if the
helicopter is 30 ft above the water?
   A) 6900 ft · lb   B) 7800 ft · lb   C) 6060 ft · lb   D) 1100 ft · lb

74) A construction crane lifts a bucket of sand originally weighing 150 lb at a constant rate. Sand is lost   74) _____
from the bucket at a constant rate of 0.5 lb/ft. How much work is done in lifting the sand 50 ft?
(Neglect the weight of the bucket.)
   A) 6250 ft · lb   B) 7500 ft · lb   C) 6875 ft · lb   D) 8125 ft · lb

75) A vertical right circular cylindrical tank measures 20 ft high and 10 ft in diameter. It is full of oil   75) _____
weighing 60 lb/ft^3. How much work does it take to pump the oil to the level of the top of the tank?
Give your answer to the nearest ft · lb.
   A) 1,884,956 ft · lb   B) 942,478 ft · lb   C) 15,708 ft · lb   D) 3,769,911 ft · lb

76) A vertical right circular cylindrical tank measures 28 ft high and 8 ft in diameter. It is full of oil   76) _____
weighing 60 lb/ft^3. How long will it take a (1/2)–horsepower (hp) motor (work output
275 ft · lb/sec) to pump the oil to the level of the top of the tank? Give your answer to the nearest
minute.
   A) 72 min   B) 143 min   C) 4299 min   D) 23 min

77) A swimming pool has a rectangular base 12 ft long and 24 ft wide. The sides are 5 ft high, and the   77) _____
pool is half full of water. How much work will it take to empty the pool by pumping the water out
over the top of the pool? Assume that the water weighs 62.4 lb/ft^3. Give your answer to the
nearest ft · lb.
   A) 84,240 ft · lb   B) 112,320 ft · lb   C) 224,640 ft · lb   D) 168,480 ft · lb

78) A conical tank is resting on its apex. The height of the tank is 8 ft, and the radius of its top is 4 ft.   78) _____
The tank is full of gasoline weighing 45 lb/ft^3. How much work will it take to pump the gasoline to
the top? Give your answer to the nearest ft · lb.
   A) 24,127 ft · lb   B) 12,064 ft · lb   C) 193,019 ft · lb   D) 3016 ft · lb

79) A tank is designed by revolving the parabola $y = 2x^2$, $0 \le x \le 2$, about the y–axis. The tank, with   79) _____

dimensions in meters, is filled with water weighing 9800 N/m^3. How much work will it take to empty the tank by pumping the water to the tank's top? Give your answer to the nearest J.
   A) 1,313,605 J   B) 656,802 J   C) 3,940,814 J   D) 41,050 J
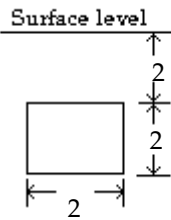
Find the fluid force exerted against the vertically submerged flat surface depicted in the diagram. Assume arbitrary units, and call the weight–density of the fluid w.

80)   80) _____

A) $\dfrac{250}{3}w$ B) $\dfrac{625}{6}w$ C) 125w   D) $\dfrac{125}{2}w$

81)   81) _____
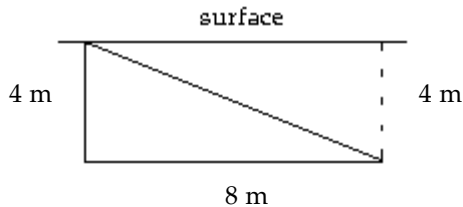

A) 4w   B) 12w   C) 24w   D) 48w

Solve the problem.

82) One end of a pool is a vertical wall 13 ft wide. What is the force exerted on this wall by the water if   82) _____
it is 4 ft deep? The density of water is 62.4 lb/ft^3.
   A) 6490 lb   B) 1620 lb   C) 3240 lb   D) 13,000 lb

83) A right triangular plate of base 8 m and height 4 m is submerged vertically, as shown below. Find   83) _____
the force on one side of the plate. (w = 9800 N/m^3)

   A) 100,000 N   B) 420,000 N   C) 310,000 N   D) 630,000 N

84) Find the force on one side of a cubical container 5 cm on an edge if the container is filled with   84) _____
mercury. The density of mercury is 133 kN/m^3.
   A) 8.3 N   B) 8300 N   C) 330 N   D) 1.7 N

85) A semicircular plate 4 ft in diameter sticks straight down into fresh water with the diameter along   85) _____
the surface. Find the force exerted by the water on one side of the plate.
  A) 249.6 lb   B) 748.8 lb   C) 332.8 lb   D) 499.2 lb

86) A rectangular swimming pool has a parabolic drain plate at the bottom of the pool. The drain plate   86) _____
is shaped like the region between $y = \frac{1}{2}x^2$ and the line $y = \frac{1}{2}$ from $x = -1$ to $x = 1$. The pool is 10 ft

by 20 ft and 8 ft deep. If the pool is being filled at a rate of 200 ft^3/hr, what is the force on the drain
plate after 5 hours of filling? Round your answer to two decimal places if necessary.
  A) 611.52 lb   B) 195.52 lb   C) 208 lb   D) 97.76 lb

1) D
2) A
3) A
4) D
5) D
6) A
7) B
8) C
9) D
10) D
11) D
12) B
13) B
14) B
15) B
16) D
17) B
18) B
19) A
20) D
21) C
22) B
23) A
24) A
25) B
26) A
27) D
28) C
29) A
30) C
31) D
32) A
33) D
34) A
35) D
36) B
37) D
38) C
39) D
40) C
41) B
42) D
43) C
44) C
45) A
46) C
47) C
48) D
49) D

50) B
51) B
52) A
53) B
54) C
55) B
56) D
57) A
58) C
59) C
60) D
61) C
62) B
63) D
64) B
65) C
66) D
67) C
68) D
69) D
70) D
71) D
72) A
73) A
74) C
75) B
76) A
77) D
78) B
79) A
80) B
81) B
82) A
83) B
84) A
85) C
86) B